

BALTIMORE COUNTY PUBLIC SCHOOLS

OFFICE OF MATHEMATICS

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SAMPLE FREE RESPONSE QUESTIONS

AP-LEVEL CALCULUS

Written by:

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1. A particle is moving along the x-axis according to the equation

$$x(t) = t \ln(t) - 3t \text{ for } t \in [1, e^t]$$

- a. For what value(s) of t is $x(t)$ negative?
 - b. For what value(s) of t is the particle at rest?
 - c. What is the total distance travelled by the particle?
2. Consider the point $P(-2, -5)$ and the circle $x^2 - 4x + y^2 + 2y = 4$.
- a. What is the area of the region enclosed by the circle?
 - b. What point(s) on the circle is closest to point P ?
 - c. Write the equation of the lines tangent to the circle at $x = 1$.
3. Let $g(x) = x^2 - 4x + 10$, $h(x) = -x^2 + 8x$.
- a. Sketch the graph of the region bounded by $g(x)$ and $h(x)$
 - b. Find the area of the region from part (a).
 - c. Find the volume of the solid formed by rotating the region from part (a) around the x-axis.
 - d. A bug walks along the border of the region from part (a) beginning and ending on the same point. How far has the bug travelled? [only do this if arclength is still an AP-topic.]

4. Let $f(x)$ be continuous on $[-2, 2]$ with the following properties.

$$f'(x) > 0 \text{ on } (-2, 0) \text{ and } (1, 2)$$

$$f'(x) < 0 \text{ on } (0, 1)$$

$$f'(0) \text{ is not defined}$$

$$f'(1) = 0$$

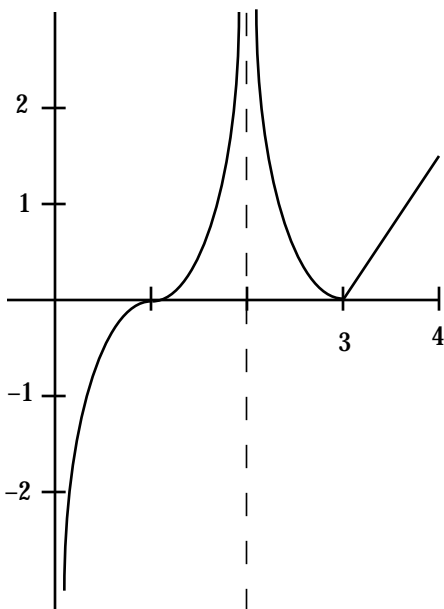
$$f''(x) > 0 \text{ on } (-1, 0) \text{ and } (0, 2)$$

$$f''(x) < 0 \text{ on } (-2, -1)$$

x	-2	-1	1	2
$f(x)$	0	1	1	2

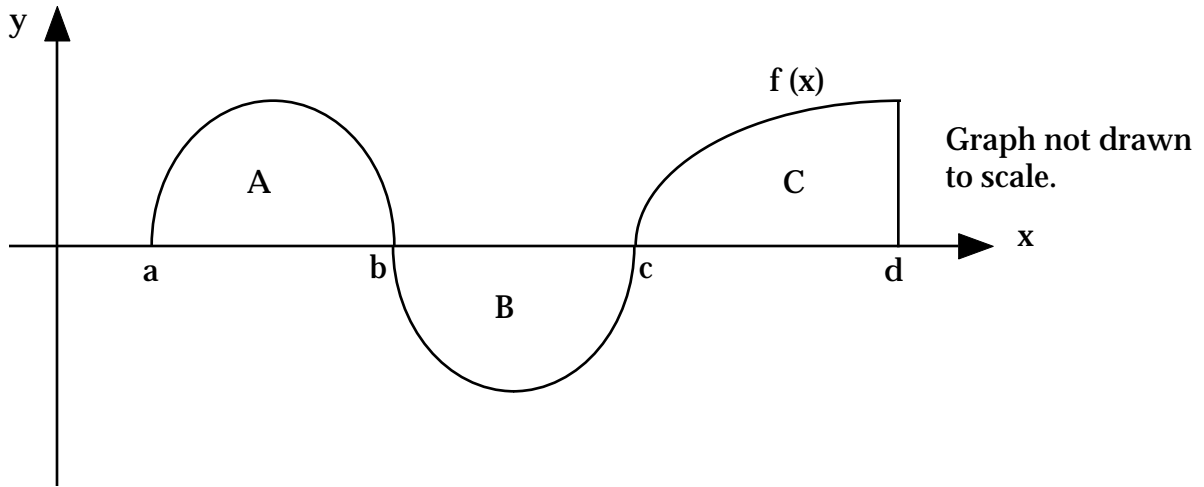
- Sketch a possible graph of $f(x)$
- For what value(s) of x does $f(x)$ attain relative or absolute extrema? Classify each as a maximum or minimum. Justify your answer.
- For what value(s) of x does $f(x)$ have a point of inflection? Justify your answer.

5. The graph of $f'(x)$ is shown below; $f(1) = 1$ and $f(3) = 1$



- Sketch a possible graph of $f(x)$ on the same axes.
- Identify and classify any relative extrema of $f(x)$. Justify your answer.
- Identify any point of inflection of $f(x)$. Justify your answer.

6. The graph of the differentiable function $f(x)$ is shown.



The area of region A = 2.5, area of region B = 2, area of region C = 2.8

a. Find $\int_b^d f(x) dx$

b. Find $\int_d^a f(x) dx$

c. Let $g(x) = f(x) + 1$, and $(d - a) = 4$

Find $\int_a^d g(x) dx$

d. Find the average value of $g(x)$.